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ABSTRACT

Within the context of the developments in mathematics and computer science, this paper argues that the emergence of the microcomputer gives mathematics the opportunity to survive as a viable and healthy discipline. Section 1 traces the development of the computer and the position of established mathematics on the sidelines of this development. Section 2 discusses some ironies in the synchronous development of computers, with their potential for transforming mathematics education via an emphasis on the computational content of mathematics, and the development of the Bouraki movement in mathematics, which stressed the syntax and structure of mathematics. Section 3 describes the Bouraki years (1945-1975), which focused on abstract formalism coupled with an exclusive mentality, and resulted in alienated client departments, aggravated employment problems, and isolation of the discipline. Section 4 considers the roots of mathematics, looking at three schools of classical mathematics and modern theoretical developments. Section 5 proposes a tetrahedron model of mathematical sciences, in which pure mathematics stands at the apex and applied mathematics, statistics, and computer science stand at its base, and sections 6 and 7 look at the implications of this model for curriculum development with respect to the three classical schools of mathematics. Finally, section 8 suggests ways of revitalizing the mathematics curriculum through, for example, using the computer to provide a graphical or computational component in every math course. (AYC)

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NEW DIRECTIONS IN TWO-YEAR COLLEGE MATHEMATICS

Sloan Foundation Conference

11-14 July 1984

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IT'S GOING TO HAPPEN ANYWAY...

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SUMMARY

The first electronic digital computer was invented 42 years ago by a math and physics teacher who was motivated by the wish to solve a system of equations arising from a problem in science. The Bourbaki movement came to life a few years later, motivated by the wish to emphasize the logical and structural aspects of mathematics. Mathematicians were captivated by Bourbaki, but were not so taken by the computer. It is time to make a serious move in education away from Bourbaki formalism and toward the applied and computational.

The computer began a transformation of society that the microcomputer will accelerate. Mathematics has been affected by the computer revolution, and will go through profound changes. Mathematicians can continue to engage in timid or delayed interaction, or they can take leadership roles.

A historical and philosophical sketch is used to construct a conceptual framework for an emerging symbiosis of the mathematical sciences and the microcomputer.

Two-year colleges have been typically more able than four-year colleges or universities to strike out in new directions. Here is a golden opportunity for two-year college math (science) departments to put this flexibility to good use, to go beyond responding to a current need. Our historical experience with information-processing devices (books, TV) and recent experience with main-frame computers will help anticipate some of the changes in store for mathematics and society.

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"It's Going to Happen Anyway . . . "

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NEW DIRECTIONS IN TWO-YEAR COLLEGE MATHEMATICS

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1. *HOW IT BEGAN* In the mid-1930's, a sequence of events was unfolding at Iowa State College that were to culminate in a remarkable invention. A professor of mathematics and physics was studying the polarizability of helium in an electric field. The model for this experiment turned out to be a wave equation. As usual for such partial differential equations, the only feasible method of solution was numerical. This gave rise to a fundamental problem, that of solving a system of linear algebraic equations. The challenge of solving such systems with something faster than the available mechanical devices became his Archimedean problem. He became obsessed with designing a machine. Two years later, in 1938, he had his *Eureka* experience. Four basic concepts for the modern computer had crystallized in John V. Atanasoff's mind. He would build a *binary, electronic* machine that used *logic systems* (not enumeration) for processing, and *automatically refreshed* capacitors for memory units. Atanasoff's fertile brain, complemented by his own skillful hand and the assistance of graduate student Clifford Berry, built the world's first electronic, digital computer - the ABC. This precocious desk-size bantling was unveiled in October 1942. The ABC could solve up to 29 equations in 29 unknowns. It was produced on a

miniscule budget, and at a locus far removed from the giants of computing and research -- in an Iowa State College physics lab.

Soon after, what today appear to be electronic dinosaurs roamed the land. It began with the ENIAC in 1944, evolved into the early UNIVAC's, and climaxed in the potent, time-sharing IBM Systems 360 in 1964. The 360 (later, 370) became an industry standard.

Ten years later, the tiny INTEL 8080 four-bit chip was to power the small mammals that would challenge the outsized lizards. MITS, a small company in Albuquerque, N.M., produced the Altair microcomputer, a \$500 kit based on the 8080 chip. By 1975, over 600 Altairs had been sold, mainly to hobbyists. In 1976, Jobs and Wozniak built their first Apple Computer, and the rest is history *au courant*.

And where was established mathematics during all of this? Mainly, on the sidelines. Established mathematicians responded to the challenge with the alacrity of a garden slug. The 1981 CBMS report, *Undergraduate Mathematical Sciences* [10] has some trenchant observations on changes since 1975: "Access to computers is up sharply, but the impact of computers has changed little" "Computer course enrollments have exploded and outnumber those in calculus."

In 1973-83, M.I.T. showed a decline in math undergraduates from 18% to 6%, while engineering went from 38% to 75%, with 35% of the undergraduates majoring in EE/CS. Many other statistical tidbits about academe could be cited.

The general public is also responding, with microcomputer sales estimated at ten million in 1984. There are likely to be 20 million sold in 1985. Microcomputers have fanned out from their toy-hobby status. A major psychological barrier was breached three years ago with the appearance of the IBM PC -- it was now OK for serious business types to own microcomputers. Just recently, IBM came out with a 32-bit desk-top System 370 at about one fiftieth the cost and size of its progenitor.

It is too late for mathematics to play its traditional role of the pacesetter in these new developments. However, the microcomputer does give us a second chance to join the parade [3]. The microcomputer has emerged as a significant new force - it is going to transform society with us, or without us. The engineers recognize the microcomputers are important. Students and the general public know they are important. Do we . . .? The survival of mathematics as a healthy, viable discipline depends on the answer to this question. We can go the way of the classics and philosophy, or we can absorb the little devil and grow.

2. *SOME IRONIES* About the time the computer surfaced, specialization was becoming a way of life for scientists and (especially) mathematicians. Yet, Atanasoff was a generalist. He had a BS in electrical engineering from the University of Florida, an MS in mathematics from Iowa State, and a PhD in theoretical physics from the University of Wisconsin. (Even today, at 81 years old, Dr. Atanasoff urges students who are interested in computer science to study all the mathematics they can.)

The microcomputer revolution, with the MITS opening shots in 1974, has been mainly the province of free-spirited, independent-minded young men. These attributes are an important part of the traditional stereotype of a creative mathematician.

The third irony is the most important for the purposes of this paper. In 1942, in Ames, Iowa, Atanasoff invented what might be considered a mathematician's dream -- a device for fast, accurate calculation. Its creation was motivated by a problem that originated in natural science. It had the potential to transform mathematics education and practice via an emphasis on the *computational content* of mathematics. Two years later, in Nancy, France, a group busied itself with a quite different activity, something that G. H. Hardy would have considered a dream. Its creation was motivated by a wish to clarify, purify and unify mathematics. It had the potential to transform mathematics education and practice via a stress on the syntax and structure of mathematics. This conglomerate product of elite French thought was known as *Bourbaki*.

Bourbaki soon jumped the Atlantic and exerted a powerful influence on established American mathematics. This led to over 25 years of narrowness that historians will view as an anomaly. They will wonder why a supposedly pragmatic people preferred Bourbaki almost to the exclusion of Atanasoff.

3. THE BOURBAKI YEARS: 1945-1975 Bourbaki influence rose very rapidly in the major universities and then filtered out and down to the rest of the mathematics community. During the three decades after World War II, established mathematics had become dangerously imbalanced. The

overstressing of abstract formalism, coupled with an exclusive mentality, alienated our client-departments, aggravated employment problems, and left us in splendidly pure isolation. Classical applied mathematicians had to take refuge in institutes, or work out of closets. The computer infant faced indifference or aversion and so went in the direction of the more friendly business and engineering departments. Statistics remained in usual quarantine. Operations Research took root in management and industrial engineering.

Service courses slipped out of our hands, while engineers and others surreptitiously, or even openly, taught math courses. (A 1981 CBMS report [10] on two-year colleges notes that since 1970 math courses taught outside of math departments have *tripled*.) Bachelors and Masters degrees tended to be viewed as way stations on the path to a pure PhD.

In 1970, the year that young people first warned the nation that it was in environmental trouble, the employment figures gave mathematics explicit warning of impending tribulations. PhD's in the most rarified of fields were being turned out by the hundreds, with no takers. Many were as pure as the vestal virgins, unsullied by any exposure to computer sciences, operations research, statistics, or even classical applied math. Did no one smell the smoke or sound the alarm ...?

In 1965, Morris Kline assailed the *New Math*, a K-12 offspring of Bourbaki, as a peril to U.S. scientific progress [5]. He has continued the assault on this symptom of the imbalance in academic mathematics.

In 1966, there were two symposia on the Impact of Computing on Undergraduate Mathematics Instruction [9]. A. Ralston gave this summary: "The only remarkable thing about the impact, thus far, of computing on the undergraduate mathematics instruction, is its absence." (A 1981 CBMS report has this to say about computer usage in math courses in two-year colleges: "The small impact of computers in mathematics teaching can be seen by noting that less than 2% of all sections of mathematics reported the use of computer assignments for students." [10, p. 88].) Ralston makes an equally interesting observation about panelist N. Macon's comment on employability: "Macon makes the *controversial* prediction that the current rate of increase of mathematics undergraduates may result in an oversupply of bachelor level mathematicians by the mid-1970's." The italics were added to emphasize that even the outspoken Ralston thought such a sensible prediction was controversial. If one turns to the report on the symposia [9, p. 670], the following remark is found: "Three panelists and several audience members took exception to the inference in Macon's talk that an oversupply of mathematicians might be produced by the mid-1970's if present growth rates for mathematics majors continue." It was lucky that Macon was not so prescient or bold as to predict an oversupply of MA's or PhD's -- he might have been hooted off the panel. This was the Bourbaki euphoria that permeated mathematics in the 1960's.

In 1967, J. Kemeny predicted that learning to use a computer would be as important as learning to read and write.

The polemics or warnings gradually built up, peaking in 1971-73. An article by Gaskell and Klamkin in 1974 documented the shortcomings of the

modern curriculum [4]. They decry the lack of attention paid to problem recognition, formulation, and follow-up. Their article made it embarrassingly clear that math graduates at all levels had to be retrained in computer science, engineering or statistics before they were of any value to industry. They noted that "Modern trends have even narrowed the field of vision, which may be necessary for true research efforts, but is a handicap for the vast majority of our students who must compete with broadly trained individuals from other fields."

Another article that ought to be mentioned, written by the current MAA director, has the devastating metaphorical title: *England was lost on the playing fields of Eaton* [11].

The job situation was so bad in the 1970's that one report in the AMS *Notices* slyly dropped non-PhD's from the unemployment figures. Chairmen dreaded the unsolicited letters from young, trusting mathematicians, who had no forewarning of what was in store for them. The PhD candidates were naive, but not fools. As the market feedback made its appearance, students began to jump ship.

How did mathematics get itself in such a fix? As the above comments show, it had cut itself off from its applied component. It had a bad case of Bourbaki-itis. Even now, the responses we observe seem to be on an *ad hoc* basis. Something like: "A dash of applied math will make our degrees more applicable, so let's give it to them." I would suggest that no counterpart of the medical needle, nostrum or knife will cure the disease. If we are going to do any more than treat symptoms, we must turn to *regimen*.

And before regimen is begun, we must seek the causes -- which means to look at our roots. What led mathematicians down the narrow road whereby abstract, formalist disquisitions -- *pure math* -- became the only value yardstick by which mathematical activity is measured? And what can be done about it...?

4. THE ROOTS OF MATHEMATICS All mathematics (for the purpose of this paper) will be divided into three parts: That of Euclid-Plato, that of Archimedes, and that of Pythagoras. The easiest to trace, cleanest to define, and most influential is the first.

The *Euclid-Plato school* is the direct antecedent of what is known as pure mathematics. It has been refined, but has not changed in any essential way. It begins with a substrate of logical axioms and immediately proceeds to more "mathematical" assumptions or *postulates*, a small set of related statements that involve a few undefined terms or *primitives*. The logic and strict rules of inference yield further statements, or *theorems*. The creation and study of such *Formal Deductive Systems* (FDS) is the quintessence of pure mathematics. It is not necessary to give an interpretation to the primitives, and is usually considered undesirable. It was rather difficult not to give an interpretation of the first and paradigmatic FDS, Euclidean geometry. (There are some scoffers who think that the content was there because of experience and the abstraction came later.) In any case, later systems, such as the non-Euclidean geometries indicate that it is legitimate to consider the FDS

as an abstract, idealistic, crystal-like product of the human mind, seemingly independent of sense data. It and its philosophical counterparts were kept alive by scholastics and thinkers through the ages. When we reach Newton (born 1642), we see the FDS applied to natural science. The primitives are interpreted, which is to say they are given meaning - as in *mass* or *length* - and the postulates become true or false, as do all of the theorems. Thus, the FDS serves as a model, and these most sophisticated of all models are known as *theories*. (Galileo, who died in 1642, had been the first to work with *algebraic* models, but they were equations, not FDS's.)

Logical rigor began to wane as the analysis based on the Leibniz-Newton calculus took hold, and mathematics drifted away from its pure state. Cauchy, Weierstrass and others came in as a cleaning crew, and then it was on to the pure and phantasmal creations of Cantor (1879). A revitalization of the foundations occurred at the turn of the century, at the hands of four mathematical logicians: G. Frege, G. Peano, A. N. Whitehead, and B. Russell. It culminated in the monumental Russell and Whitehead "Principia Mathematica" (1910), which purported to reduce all of mathematics to logic. The last important actor in this drama was David Hilbert, who characterized mathematics as a game played with meaningless signs and subject only to the constraints of consistency. Peano and Hilbert's formalistics are lumped with the others' logistic, since the differences are much less important than what they have as a common foundation, the Formal Deductive System.

And so, on to Bourbaki. This school of mathematics, still dominant today, has the following attributes. Its objects of play are abstract, formal, ideal. It is concerned with syntax and structure. And (to protect Hilbert from a great man's indiscretion) the primitives are uninterpreted. Semantics, content, truth or falsity -- all are irrelevant. As for existence, *it stands for nothing more than consistency*. Even the self-consciously pure mathematicians, such as G.H. Hardy, do not always deal with Formal Deductive Systems. However, the FDS is their paradigm, and they maintain a high level of logic with no interpretations as crutches. This (almost) completes the story of the Euclid-Plato school.

The Archimedean school can be thought of as a point of view that began with Archimedes, the father of (classical) applied mathematics. He was familiar with FDS's, but chose not to restrict himself to them. He used graphics, inferential arguments, and (probably) physical models. Kline has referred to him as the greatest mathematician of antiquity. His approach to mathematics has been used through the centuries by scientists and mathematicians, among them the great Euler. A repudiation of such relatively informal methods began with Lagrange (1797), who declared his independence of graphs. Many engineers, physicists, applied mathematicians, and others employ Archimedes' mixed methods.

The Pythagorean school was well-defined in antiquity, but its trace through history is fuzzy. Pythagoras' outlook went beyond Archimedes'

in that he wished to apply mathematics to both the natural external world and one's internal (spiritual) world. Thus, Pythagoras' view of mathematics had a subjective-mystical content that was absent from the other two schools. On the other hand, it had a restrictive view of acceptable mathematics. First, there was an insistence on construction and second, the mathematics was restricted to integers or ratios of integers. (The discovery of irrationals was seen as a threat...)

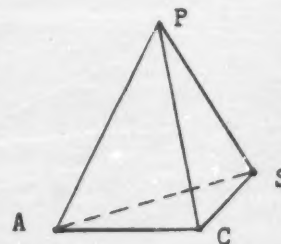
If one journeys in time to the late 1800's, the world of L. Kronecker and L.E.J. Brouwer, it is perhaps not too far-fetched to see Intuitionism as a distant relative of the Pythagorean School. The Intuitionists had a subjective-mystical component that was not shared by the classical practitioners or the Cauchy-Weierstrass clean-up crew. On the other hand, they were logically more restrictive. For one thing, they rejected the law of the excluded middle. Moreover, they insisted that integers were the ground state of mathematical reality and were scandalized by the free-wheeling antics of Cantor.

Moving into the 1960's, we find the Constructive Math of Errett Bishop. Although the mysticism is gone, a subjective aspect remains. In fact, Bishop claimed there was no way to capture the content and meaning of mathematics via formalistics. Logic is employed at the common (mathematical) sense level, which is learned by *doing*. However, proofs are *more* demanding than the other two schools. This is due to a critical difference in the concept of *mathematical existence*. The usual

view is existence means consistency, or freedom from contradiction. For Bishop *there exists B* means that you can construct *B*. Thus, indirect proofs, or proofs by contradiction are not acceptable when the hypothesis asserts the existence of something. Such efforts might be looked on as a pointer, or a way to increase one's degree of confidence that a proof -- or construction -- might be discovered. Since the constructivist approach divorces many cherished results from theoremhood, it is easy to see why it is not very popular.

5. THE MATHEMATICAL SCIENCES J.W. Kennelly of Clemson University was perhaps first to use the designation, in 1971, *Department of Mathematical Sciences*. The term is unwieldy and somewhat inaccurate -- since it does not mean that mathematics has switched from an inferential to an empirical discipline. But it was (and is) an important signal that the prevalent narrow confines were being rejected in favor of a broader, eclectic view of the field.

In 1974, I suggested a tetrahedron model for the math sciences concept. At the apex is Pure mathematics above a base of (classical) Applied math, Statistics, and Computing. (In my original presentation C denoted *computer science*, since I was imbued with the naive hope that math departments would recognize its mathematical content and adopt the teen-ager as a member of the family.)



Different subfields can be represented by various regions. Thus, probability might be represented by a band running from P to S, while operations research might be indicated by a strip going from A to S. The apex is sharp and clean. The base makes a fuzzy merge with the empirical world.

6. CURRICULUM CLASSIFICATION The established divisions of *Algebra*, *Analysis*, *Geometry-Topology* (with an occasional fourth one, *Applied*) reflect a specialist's slant, and are useful at the level of university research. Perhaps we in the colleges were -- and are -- overly influenced by this one-useful classification.

Kemeny, Snell and Thompson, with their pioneering "Finite Mathematics" (1956) thrust discrete mathematics, with interesting and important applications, into the freshman offerings. F.S. Roberts' "Discrete Mathematical Models" (1976) appears almost as a two-decade commemorative. Nevertheless, in 1984, there are some who feel the need to keep the discrete at bay. For reasons to be elaborated below, I suggest that *continuous* and *discrete* be accepted in our curriculum thinking as fundamental classifications.

7. THE MATHEMATICAL SCIENCES AND THE THREE SCHOOLS OF MATHEMATICS

The tetrahedron model will be expressed very differently in a curriculum, depending on which school dominates our outlook. Let us return to each school.

The Pure (Euclid-Plato, Bourbaki) approach provides precision, structure and a logical basis. These properties strongly suggest that there is also an accompanying certitude, but here is where we must be careful. Some of the arrogance that is sometimes attributed to mathematicians might have its origins here. Yet, Godel's 1931 result tells us that any Formal Deductive System that is adequate for even *just arithmetic* is either inconsistent or incomplete. So the FDS will be logically flawed or else incapable of proving a statement that (under our interpretation) is known to be true. Godel tells us there are *inherent limits* to what pure mathematics can do. Somehow, we do not seem to be able to accept this result. It almost seems as if the attitude is: "If we don't think about it, maybe it will go away." A look at a parallel situation in physics, bolstered by a trenchant observation of B.B. Mandelbrot might be instructive.

In studying or teaching the history of science and math, one notes that 1879 is surrounded by important happenings. That year marks the death of Maxwell and the birth of Einstein, both men being associated with the shift from action-at-a-distance to the *field* concept. A new physics was definitely on its way. It was turbulent, exciting, and filled with healthy controversy. About the same time, as conventional history would have it, mathematics was witnessing a young, creative, freedom-loving Cantor being harassed by some cranky old-timers. Cantor and the free-spirits won out. So math also went through turbulent, exciting times, but filled with disturbing controversy.

Although I knew there was something awry with turn-of-the-century mathematics, I was never able to articulate it. This was not just a question of controversy - a basic shift had occurred which I could not explain. B.B. Mandelbrot gave a brilliant lecture on fractals at the University of

Florida (where I was on sabbatical) in 1981, and suddenly the puzzle fell in place. The gist of his remarks appear in his "Fractals and the Geometry of Nature", 1982 [7].

On page 3 he quotes F.J. Dyson's summary of his theme:

'Modern mathematics began with Cantor's set theory and Peano's space-filling curve. Historically, the revolution was forced by the discovery of mathematical structures that did not fit the patterns of Euclid and Newton.' ... 'The mathematicians who created the monsters regarded them as important in showing that the world of pure mathematics contains a richness of possibilities going far beyond the simple structures they saw in Nature. Twentieth-century mathematics flowered in the belief that it had transcended completely the limitations imposed by its natural origins. Now, as Mandelbrot points out, Nature has played a joke on the mathematicians. The 19th-century mathematicians may have been lacking in imagination, but Nature was not. The same pathological structures that mathematicians invented to break loose from 19th-century naturalism turned out to be inherent in familiar objects all around us [via fractal analysis].'

On page 1, after commenting on the high degree of complexity exhibited by Nature, Mandelbrot writes, "Mathematicians have disdained this challenge, however, and have increaaingly chosen to flee from nature by devising theories unrelated to anything we can see or feel."

The term that occurred to me as I heard a similar message from Mandelbrot in the University of Florida auditorium was *humanistic conceit*. While the physical scientists were learning from nature, Cantor and modern mathematics were on a narcissistic intellectual trip.

There is a second interesting parallel between physics and math. In 1824, S. Carnot published his gem on heat engines. Physicists were faced with a strange new law (formulated a few years later by R. Clausius as the

second law of thermodynamics) - there were limits set by nature on what an engineer or physicist could do. No matter how clever he was, how ingenious his design, how superior his materials or fuel, any heat engine operating between the temperatures T_{lo} , T_{hi} had an upper bound on its efficiency, $(T_{hi} - T_{lo})/T_{hi}$. Scientists had bumped up against a strange and baffling limit, but they accommodated their thinking to it. About a hundred years later, they came across an even more baffling limit, W. Heisenberg's Uncertainty Principle (1927). The observer had an upper bound on the accuracy of paired observations (such as displacement and velocity). As one became more accurate, the other became less accurate. The startled physicists recovered, absorbed this second big lesson from nature, and went on. (It is ironic that about the same time, Hilbert was heralding the power of formalistics.)

Now contrast this with how mathematics reacted to Godel's 1931 limiting (meta) theorem. It was so deep that it took years for even experts to absorb, but by 1945 its conclusions were understandable to professional mathematicians. And what were its effects? Established mathematics went on as if it had never happened -- the FDS was still emperor.

Being an expert in Pure math does not turn one into a god-like creature. Like all other man-made systems, it has its shortcomings and flaws. As depicted on the tetrahedron model, it is simply one component of the mathematical sciences. It might be pedagogically unwise to expose a logical flaw to our young charges. However, if we have a gut-level awareness of this limitation, we might come across to our students and non-math colleagues, as less remote (and less omniscient).

The Archimedean approach, essentially that of the "working" mathematician,

seems to fit comfortably in the math sciences scheme.

Now for the third, distantly Pythagorean school. It has no explicit locus on the tetrahedron, and yet it is of great importance because of the way it can modulate how we teach mathematics.

It's entree is through vertex C, the computing or computational aspects. Henry Cheng has addressed this point in his *Constructive mathematics and computer science*, 1972 [2]. He notes that one of the main aims of Bishop's Constructivism is to inject the theorems of conventional math with computational content. On the other hand, the constructive approach restricts itself to finitely performable sequences of operations. This algorithmic approach allows a tie-in with an ALGOL (or other high level language) program. Thus the algorithm is a link between computer science and mathematics.

At every level and in every course, the question needs to be asked: Are we emphasizing computational aspects? Have we explored the computational aspects of the method or theorem? If it is an indirect proof, is there a constructive proof available? Are we choosing errors, bounds or tolerances as reals, when they could simply be integers or reciprocals of integers?

8. *THE COMPUTER AND THE CURRICULUM* The two-year colleges serve a significant number of students in higher education, approximately one-third of all college enrollments. Many of the students who enter college this fall will graduate in 1986 with an AA degree in hand. In that year approximately 30 million microcomputers will be sold. Vocational-technical, career, or liberal arts students -- it does not matter -- they will face a profoundly different society from their counterparts of just two years ago. The electronic marvels of their world will operate on principles very much like the 1942 desk-size ABC, plus a stored program. But there will be no punched-card

input/output, only the familiar keyboard, printer and high-resolution screen. It will have massive external storage, about a million characters on each pocket-size disk, and an equal amount of internal memory. It will be able to do any of the scientific computing, word processing, and perhaps even *symbolic* manipulation of machines that in the 1970's cost up to \$50,000. In fact, the electronic marvels of the 1970's will appear like bulky museum pieces to the younger graduates.

The AA's who transfer to engineering schools will find that hundreds of them will require all students to have their own microcomputers. The graduating AA's will be on the doorstep of an information-based society. As W.F. Lucas of Cornell has pointed out, "The impact of the computer is much, much bigger than most of us can conceive." [6]

There is an emerging symbiosis of *mathematics* and computer science. To date *mathematicians* have only been marginally involved. The microcomputer now gives us a second chance. It will not do to merely *cope* with the computer, or view it as a nuisance that must be contained. Just as a text, dictionary or thesaurus is a tool for augmenting the reach of the mind, so too, is the microcomputer such a tool, a dynamic, interactive one.

Plans need to be made now for making the computer a part of every math sciences course. Every course in the kind of mathematics envisaged here would have a graphical or computational component. If the formal aspects of the subject, such as geometry (or topology) are not of a numerical nature, the intuitive aspects can be strengthened by graphing. We will be able to do some very important things that we now merely talk about: Numerical integration and composition of curves (Calculus), graphing fields of lineal elements and qualitative behavior (D.E.), and operations with realistically large

matrices (Linear Algebra).

Colleges are responsible to see that the 1986 graduate will have an experience in computers. For math science students, that minimal course ought to be a programming course that *emphasizes structured design and problem-solving*. If only get-the-code-out-the-door courses are available, math sciences needs to teach its own. (If the department has so removed itself from computing that such a course is not seen as part of the math sciences, it might be better to use only packaged software.)

Departments that have proudly maintained that "...mathematics requires only the human mind plus paper-and-pencil..." need to start singing a different refrain -- quickly. We need to convince administrations that the *math sciences need equipment budgets*. A good argument might be: *Microcomputers cost no more than microscopes -- which gives more educational return for the money?* Administrators, weary of the endless budget-gobbling of biology departments, might respond to this one.)

There are now many one-day to two-week workshops or seminars designed for faculty who want to learn topics in computing. Many are relatively cheap. Currently, only a *small* proportion of those who take advantage of these are mathematicians. However, AMATYC and the MAA have been offering more experiences in computer-related topics each year.

Available software has hitherto been a weak link in efforts to infuse the computer in the math sciences curriculum. Sometimes code (usually unstructured BASIC or FORTRAN) was pasted into standard math texts. However, there are some materials available in which text and software are interwoven. Most of us in teaching do not have the time or expertise to produce the software needed. The authors who are producing these classroom-ready materials are playing a vital role in the evolving role of the math sciences.

Integrated Software

Following is a short list of works in Calculus and Linear Algebra

Garry Helzer (Univ. of Maryland), "Applied Linear Algebra with APL", Little Brown, Boston, 1982. (*)

D.L. Orth (IBM, NY), "Calculus in a New Key", APL Press, Palo Alto, Calif., 1976. (*)

Gareth Williams (Stetson Univ., Florida), "Applied Linear Algebra with Applications", Allyn & Bacon, Rockleigh, N.J., 1984.

Gareth & Donna Williams, *Linear Algebra Computer Companion*. This is an interactive BASIC software package. It is on two diskettes formatted for the Apple II.

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F. Hickernell & W. Proskurowski (Univ. of Southern California), Calculus (DEC Gigi).

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